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Stability of the Boundary Layer on a Swept Wing with Wall Cooling

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Linear stability theory was used to examine the propagation of laminar instabilities in the leading edge region of a transonic swept wing with wall cooling. The condition of real group velocity ratio on the eigenvalue computation was examined for the case of crossflow instabilities. When temporal theory was used, this condition resulted in a single wave of maximum amplification. When spatial theory was used, and for a limited range of angles of wavegrowth direction, the growth rate in the direction formed by the real ratio of the group velocities and the direction itself were insensitive to the orientation of the wavegrowth. It was found that wall cooling has a stabilizing effect on crossflow disturbances, but the stabilization is mild compared to the stabilizing effect that wall cooling has on Tollmien-Schlichting waves.

Nomenclature

AD = adiabatic
C = wing chord
C_p = pressure coefficient
d_i = disturbance quantity
F = frequency, Hz
j = eigensolution
k = magnitude of the wavenumber vector
M = Mach number
N = integral of amplification rates
p = disturbance pressure
Q = freestream velocity
R = Reynolds number = $U_e^* \delta^* / \nu_e^*$
R_x = Reynolds number = $U_\infty^* x^* / \nu_\infty^*$
s = disturbance temperature
T = mean temperature
t = time
U = mean velocity in the x direction
U_c = locally maximum crossflow velocity
u,v,w = disturbance velocity in the x, y, and z directions, respectively
V_α, V_β = group velocities in x and z directions, respectively
x,y,z = Cartesian coordinate system
α,β = wavenumbers in the x and z directions, respectively
δ^* = incompressible displacement thickness in the x direction
γ = ratio of specific heats
ν = kinematic viscosity
σ = spatial growth rate
ϕ = angle, $\tan^{-1}(V_\beta / V_\alpha)$
ψ = angle of the wavenumber vector with respect to the x direction
$\bar{\psi}$ = angle of the direction of growth with respect to the x direction
ψ_p = angle of the external streamline with respect to the x direction
ω,ω_r = angular frequency
ω_i = temporal amplification rate

Subscripts

ad = adiabatic
e = boundary-layer edge conditions
w = wall conditions
c = normalized with the wing streamwise chord
∞ = freestream conditions
ϕ = in the direction obtained from the group velocities
x = in the x direction

Superscripts

(*) = dimensional quantity

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I. Introduction

THE increasing cost of petroleum products and their questionable availability in the future create new conditions in the design of transport aircraft. These conditions make reduction of aircraft drag more important than it has been in the past and also justify consideration of alternative aircraft fuels.

Laminar flow control is a technology that can potentially provide significant drag reductions[1] by reducing the skin friction over a portion of the exterior surface of an aircraft. Also, recent studies[2-4] show that hydrogen is a viable candidate as an alternative fuel for powering future air transports.

Because hydrogen has to be stored in liquid form at cryogenic temperatures, the available cooling power could be used[5] to lower the temperature of the skin of part of the exterior surface of the airplane and maintain laminar flow. So, if hydrogen is used for aircraft propulsion, the available heat sink could be used to reduce aircraft drag. Then, the need to be independent of fossil fuels and the need to reduce drag are both satisfied by using hydrogen as the fuel.

The effect of wall cooling on the stability of the two-dimensional boundary layer in air has been investigated. Lees[6] used asymptotic theory and found that wall cooling stabilizes two-dimensional disturbances in a flat plate boundary layer. Mack[7,8] used mainly inviscid theory and found that wall cooling stabilizes first-mode disturbances but destabilizes the supersonic modes. Since the supersonic modes are absent in the subsonic-transonic regime, this adverse effect of cooling has no bearing in the present study.

Boehman and Mariscalco[9] computed the effect of wall cooling on the stability of the boundary layer in a shock tube and on a flat plate. Using the complete equations, they found that wall cooling has a significant stabilizing effect on two-dimensional and three-dimensional disturbances in the transonic regime. As an example, it is shown in Ref. 9 that the minimum critical Reynolds number (based on distance from the leading edge of the plate) for a two-dimensional disturbance at a Mach number of 0.9 increased from 90,000 at adiabatic conditions to over 4 million at a wall-to-adiabatic temperature ratio of 0.759. They also found that the Mach number effect on stability is small for $0.6<M<0.9$. A program developed by the author[10] was used to calculate the temporal stability of three-dimensional disturbances in a compressible Falkner-Skan boundary layer. An e^N calculation shows[10] considerable increases in the transition Reynolds number with wall cooling on a supercritical airfoil.

High transition Reynolds numbers arise from wall cooling because decreasing the wall temperature in air decreases the viscosity close to the wall which results in a thinner, more stable, boundary layer. Low-speed experiments in air[11,12] show a decrease in transition Reynolds number with wall heating, which is the opposite effect. Low-speed stability experiments in air[13] show the stabilizing effect of wall cooling both on the neutral stability curve and on the logarithmic amplitude growth of a single frequency wave.

Future transports with supercritical airfoil sections are expected to have wing sweeps lower than those currently in use. However, the wing sweep will still be high enough for three-dimensional crossflow effects to dominate the stability characteristics. This means the previous publications on the stabilizing effects of wall cooling will be inadequate for this case. A rough estimate of the sweep angle range for onset of significant three-dimensional effects is 10-15 deg.

Under these circumstances, if the attachment line of a swept wing is laminar, then transition is usually the result of unstable waves that arise due to crossflow. Crossflow is, by definition, the boundary-layer flow in the direction normal to the external streamline. A number of recent papers[14-22] examined the problem of the propagation of three-dimensional disturbances in a three-dimensional boundary layer. Most of the work arises from renewed interest in laminar flow control through surface suction.

The important question that arises in the spatial theory of these three-dimensional disturbances in a three-dimensional boundary layer is: What is the definition of the direction of propagation of the disturbance? Nayfeh[18] and Cebeci and Stewartson[19] proposed that for monochromatic waves, the direction of propagation is given by the real ratio of the complex group velocities. In the calculation of N factors,[19] this direction was maintained constant and equal to the one at neutral stability. Hence, the total effect of this condition on growth factors has not yet been evaluated.

The purpose of the present work is to estimate the effect of wall cooling on crossflow instability. However, before this can be done, the effect of computing spatial eigenvalues with a real ratio of the complex group velocities must be examined. The discussion will be confined to crossflow instability because it is already known that wall cooling has a strong stabilizing effect on Tollmien-Schlichting instability.

The problem is outlined in Sec. II. Section III describes results from eigenvalue calculations that result in a real ratio of the complex group velocities. In Sec. IV, the effect of wall cooling on crossflow instabilities is examined, and in Sec. V, the results are summarized.

II. Formulation of the Problem

The stability of a three-dimensional laminar, compressible boundary layer over a wing with variable wall temperature is considered. The wing is of infinite span, 35-deg sweep, and a streamwise chord of 2.46 m. The freestream Mach number is $M=0.891$. The wing is defined in the Cartesian coordinate system shown in Fig. 1. The x axis is in the direction of the chord normal to the leading edge, the y axis is normal to the surface, and the z axis is along the span. This wing has been used before in three-dimensional stability calculations.[15,20,21] To nondimensionalize the Navier-Stokes equations, the local incompressible displacement thickness in the x direction δ^*,

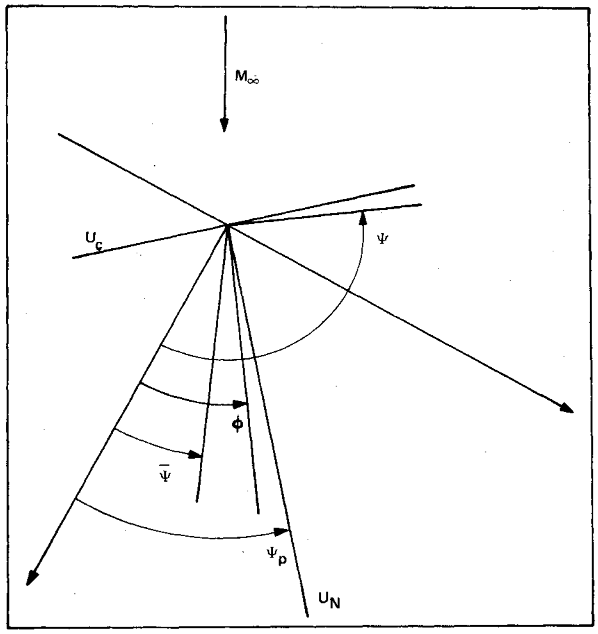

Fig. 1 Coordinate system.

the local boundary-layer edge velocity in the x direction U_e^*, and the local freestream temperature T_e^* are used.

Locally parallel flow and sinusoidal disturbances in the form of travelling waves are assumed

$$d_i(x,y,z,t)=j_i(y)\exp(\alpha x+\beta z-\omega t) \tag{1}$$

where ω is the frequency, α and β the wavenumbers in the x and z directions, respectively, t the time, $j_i(y)$ the distribution of the disturbance amplitude and phase across the boundary layer (eigenfunction), and d_i stands for the disturbance quantities. The d_i are

$$d_1=u,\ d_2=\partial u/\partial y,\ d_3=v,\ d_4=p$$
$$d_5=s,\ d_6=\partial s/\partial y,\ d_7=w,\ d_8=\partial w/\partial y \tag{2}$$

where u, v, and w are the disturbance velocities in the x, y, and z directions, respectively, p the disturbance pressure, and s the disturbance temperature.

Details of the system of equations that result from using the disturbance quantities in Eq. (2) and from linearizing the Navier-Stokes equations, as well as the method of solution of the resulting eigenvalue problem, are presented in Ref. 21. In what follows it is assumed that when temporal theory is used, ω_r and ω_i are the real and imaginary parts, respectively, of the eigenvalue, and α and β are the real wavenumbers. When spatial theory is used, σ is the growth rate along a direction $\bar{\psi}$, and α_r and β_r are the real parts of α and β that form the wavenumber vector k in the direction ψ. Also, α_i and β_i are the imaginary parts of α and β, and F is the input real frequency in Hz. Hence,

$$\psi=\tan^{-1}(\beta_r/\alpha_r),\quad k=(\alpha_r^2+\beta_r^2)^{1/2} \tag{3a}$$

$$\bar{\psi}=\tan^{-1}(\beta_i/\alpha_i),\quad \sigma=(\alpha_i^2+\beta_i^2)^{1/2} \tag{3b}$$

The group velocities in the x and z directions, respectively, are

$$V_\alpha=\partial\omega/\partial\alpha,\quad V_\beta=\partial\omega/\partial\beta \tag{4}$$

and their ratio defines the complex (in general) ray angle[18,19]

$$\phi=\tan^{-1}(V_\beta/V_\alpha) \tag{5}$$

A method of computing the group velocities is given in Ref. 21. The angles ψ, $\bar{\psi}$, ϕ (if real), and ψ_p, which is the angle of the local potential streamline with respect to the x axis, are shown in Fig. 1. The angles are measured positive in the counterclockwise direction.

III. Calculations of Crossflow Disturbances with Real Group Velocity Ratio

The direction of propagation of a wave of a certain frequency, defined[18,19] as the ray angle ϕ, is given by Eq. (5) and is real. Since the group velocities are obtained from a local calculation, this condition applies strictly to homogeneous systems. An example of a homogeneous system is fully developed pipe flow. The growing boundary layer is a nonhomogeneous system. Also, the requirement that the ratio of the group velocities be real still leaves one degree-of-freedom for the spatial problem unspecified. That is, one can compute different eigenvalues that have real ϕ by choosing $\bar{\psi}$, or β_r, as an independent variable. In Ref. 19, ϕ was kept constant and equal to the value computed at neutral stability.

The question that arises is the following: Since the boundary layer is inhomogeneous, how does one calculate N factors (which is an integrated quantity) while satisfying, locally, ϕ to be real? To answer this question, the temporal problem will be examined briefly. In the temporal problem, if one insists on keeping ω_r constant and V_β / V_α real, then the eigenvalue problem is determined uniquely. Locally $\omega = \omega(\alpha,\beta)$ only, for constant x and z, and for constant ω_r one gets

$$\mathrm{d}\omega_r = \frac{\partial \omega_r}{\partial \alpha}\mathrm{d}\alpha + \frac{\partial \omega_r}{\partial \beta}\mathrm{d}\beta = 0 \tag{6}$$

Also, because V_β / V_α is real,

$$\frac{\partial \omega_r/\partial \alpha}{\partial \omega_r/\partial \beta} = \frac{\partial \omega_i/\partial \alpha}{\partial \omega_i/\partial \beta} \tag{7}$$

Equations (6) and (7), together with $\omega_i = \omega_i(\alpha,\beta)$, lead to

$$\mathrm{d}\omega_i = 0 \tag{8}$$

which means that ω_i is at an extremum. In Ref. 15, this extremum was found to be a maximum. This fact is used in the current SALLY stability computer program[15] to calculate N factors when the "envelope" method of computation is selected.

From the preceding discussion, it can be concluded that the temporal theory provides only one way of computing N factors with ϕ being real.

Because, as was mentioned, in the spatial theory there is more than one way to satisfy ϕ being real, one would like to select the wave that, locally, is amplified the greatest in the direction of propagation. That is, the quantity

$$\sigma_\phi = -\alpha_i \cos\phi - \beta_i \sin\phi \tag{9}$$

must be at a maximum. Remembering that when ϕ is chosen as an independent variable the eigenvalue problem is uniquely defined, Eq. (9) shows an extremum at

$$\phi = \tan^{-1}(\beta_i/\alpha_i) \tag{10}$$

To arrive at Eq. (10), $\mathrm{d}\sigma_\phi/\mathrm{d}\phi$ is set to zero, using $\mathrm{d}\alpha/\mathrm{d}\beta = -\tan\phi$ and the Cauchy-Riemann relations. However, it turns out that Eq. (10) leads to $\mathrm{d}^2\sigma_\phi/\mathrm{d}\phi^2 = 0$, and the extremum is an inflection point.

The first attempt to compute spatial eigenvalues with real ϕ used a double Newton-Raphson scheme that iteratively updated the wavenumber k and the growth direction $\bar{\psi}$ in order to result in a real given ϕ for a given frequency. The scheme failed, in general, to converge for reasons which will be explained.

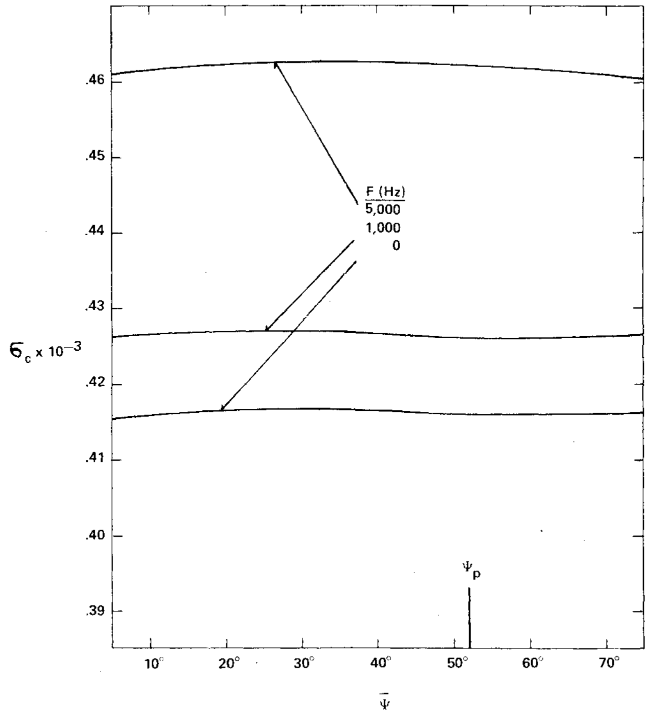

Fig. 2 Variation of growth rate σ_c with the growth direction $\bar{\psi}$ at $x/C = 0.00186$.

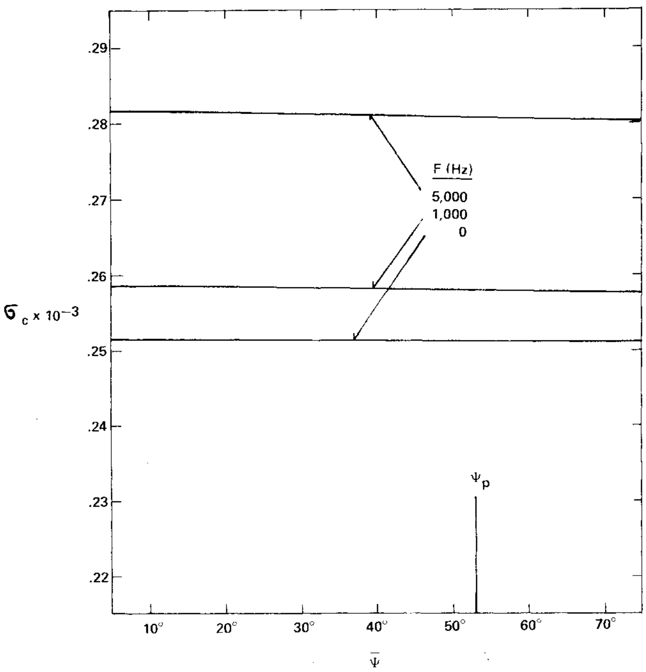

Fig. 3 Variation of growth rate σ_c with the growth direction $\bar{\psi}$ *at* $x/C = 0.01222$.

Next, $\bar{\psi}$ was chosen as a parameter and k updated so that ϕ was real. Figures 2 and 3 show the resulting σ_c, normalized with the wing chord C, for frequencies of 0, 1, and 5 kHz and for two x/C stations close to the leading edge. The Nayfeh-Padhye[17] transformation was not used because it was desired to calculate the effect of the varying group velocities. When the angle $|\bar{\psi} - \psi_p|$ was increased beyond $\sim$40 deg, the iteration scheme used failed to converge to an eigenvalue. The reason of convergence failure is not known to the author at this time. Hence, it was not possible to produce growth rates

Table 1 Eigenvalue calculations with real ϕ

	$\bar{\psi}-\psi_p$, deg	$(k\bar{C})10^{-5}$	$\psi-\psi_p$, deg	$\phi-\psi_p$, deg
		$F=1000$ Hz		
	−40	0.1853	84.45	−3.95
	−30	0.1860	84.47	−3.98
	−20	0.1858	84.45	−3.98
$x/C=.00186$	−10	0.1861	84.45	−4.00
	0	0.1864	84.46	−4.01
	10	0.1868	84.46	−4.03
	20	0.1873	84.47	−4.05
		$F=5000$ Hz		
	−40	0.1785	83.00	−3.72
	−30	0.1802	83.00	−3.70
	−20	0.1792	83.02	−3.77
	−10	0.1796	83.03	−3.79
	0	0.1800	83.03	−3.81
	10	0.1805	83.04	−3.83
	20	0.1808	83.05	−3.85
		$F=1000$ Hz		
	−40	0.1129	85.87	−3.11
	−30	0.1132	85.87	−3.13
	−20	0.1137	85.88	−3.16
$x/C=0.01224$	−10	0.1149	85.88	−3.19
	0	0.1142	85.88	−3.18
	10	0.1141	85.88	−3.19
	20	0.1152	85.89	−3.22
		$F=5000$ Hz		
	−40	0.1048	84.22	−2.92
	−30	0.1046	84.22	−2.98
	−20	0.1045	84.21	−3.02
	−10	0.1056	84.24	−3.08
	0	0.1066	84.27	−2.98
	10	0.1075	84.29	−3.08
	20	0.1081	84.29	−3.09

with ϕ real. Thus, conclusions will be drawn only for the limited range of $\bar{\psi}$ examined. The following conclusions can be drawn from these results.

The value of σ_c is almost constant, independent of the angle $\bar{\psi}$. Also, the resulting group velocities have almost identical ϕ, as shown in Table 1. That explains why the attempt to find eigenvalues by prescribing ϕ failed, because ϕ is almost constant. The range of resulting wavenumbers k is very narrow, but this range varies significantly with the chordwise coordinate x/C, as shown in Table 1. Hence, in order to compute N factors, one can choose any of these waves with very small error. For the cases examined, the error is always less than ~0.3%, a difference which may be entirely due to numerical error.

These computations were restricted to crossflow instabilities. That is, during the iteration, the wavenumber k was oriented in a direction ψ close to the crossflow direction. The resulting range of ψ is very small, as found earlier.[15,20,21] Based on these results, in what follows, N factors will be computed using $\bar{\psi}=\psi_p$ and V_β/V_α real because, as found, the waves with real ϕ and different $\bar{\psi}$ result in growth rates in the direction ϕ, which are almost identical.

IV. Effect of Wall Cooling on Crossflow Instability

To estimate the effect of wall cooling on the stability of the three-dimensional boundary layer, an existing finite-difference boundary-layer program[15] was appropriately modified so that the wall-to-freestream temperature ratio T_w/T_∞ could be specified. The enthalpy, rather than the temperature, is a dependent variable in the program. Hence, the wall enthalpy is calculated from

$$\frac{H_w}{H_e}=\frac{(T_w/T_\infty)}{(T_e/T_\infty)}\left(1+\frac{\gamma-1}{2}M_e^2\right)^{-1} \tag{11}$$

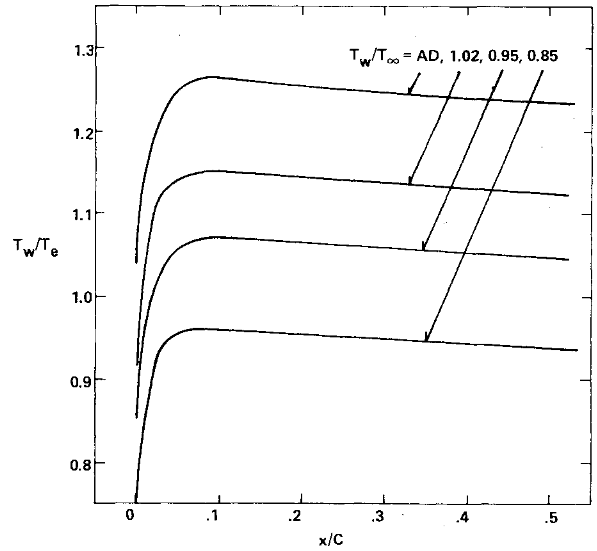

Fig. 4 Wall to local freestream temperature ratios varying with the chord location.

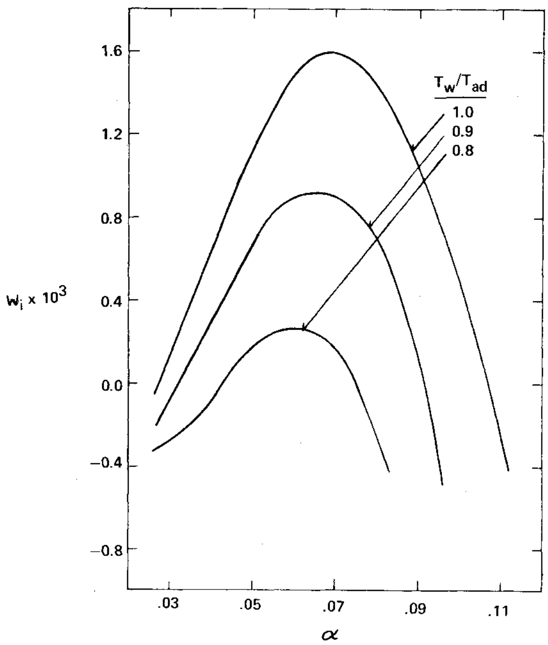

Fig. 5 Temporal amplification rates for a flat plate compressible boundary layer and for different wall temperatures.

where M_e is the edge Mach number and T_e is the edge temperature calculated from isentropic relations.

Cooling the wall in air lowers the viscosity close to the wall, which in turn gives rise to thinner boundary layers. Figure 4 shows the effect of different wall-to-freestream temperature ratios. Because these ratios were kept constant for all the x/C stations, the local wall-to-edge temperature ratios are varying with x/C. When $T_w/T_\infty=0.85$, it can be deduced from Fig. 4 that the wall-to-adiabatic temperature ratio (T_w/T_{ad}) is below 0.77. Hence, these temperatures cover a good part of the range of estimated cooling ratios to be used in design studies for a transport with laminar flow control through wall cooling.[5] However, it should be noted that the range of cooling ratios selected in Ref. 5 was based primarily on considerations of Tollmien-Schlichting instability.

As a reminder of the strong stabilizing effect that cooling has on Tollmien-Schlichting waves, Fig. 5 shows some temporal amplification rates for a flat plate boundary layer. The conditions are: $\psi=30$ deg, $M=1.6$, $R=\sqrt{R_x}=1500$,

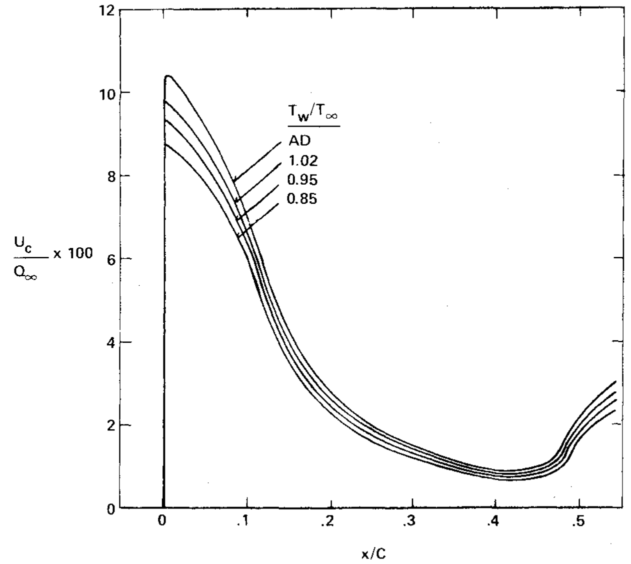

Fig. 6 Variation of maximum crossflow velocity for different wall temperatures.

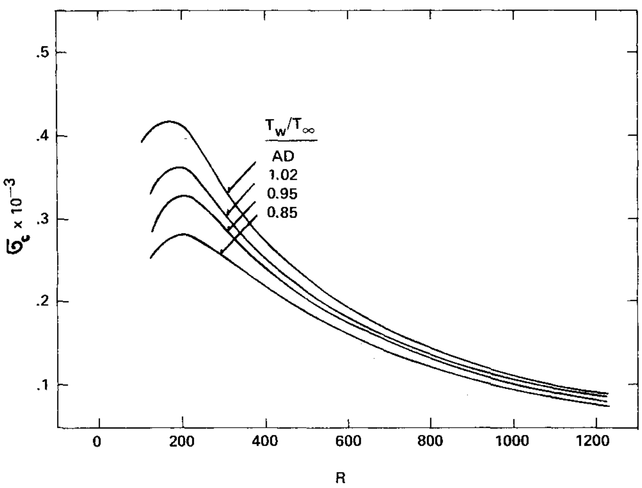

Fig. 7 Variation of growth rates σ_c with different wall temperatures for $F = 0$ Hz.

and $T_w/T_{ad} = 1.0$, 0.9, and 0.8. The nondimensionalization is based on U_∞ and $L = \sqrt{\nu_\infty x/U_\infty}$. The wavenumber α is in the stream direction. Figure 5 shows that a 20% reduction of the wall temperature from its adiabatic value reduces the maximum amplification rate by a factor of eight. This figure was produced using the full eighth-order system with the author's two-dimensional compressible stability program.

Part of the reason that wall cooling does not have a strong influence on crossflow instability can be seen in Fig. 6. The maximum crossflow velocity U_c, normalized with the freestream velocity Q_∞, is reduced, but not by very much, when the wall temperature is decreased. This confirms the suspicion expressed in Ref. 5 that cooling does not affect crossflow instability as much as it affects the Tollmien-Schlichting instability.

The distribution, near the leading edge, of the spatial amplification rates σ_c, normalized by the wing chord C, is shown in Figs. 7-9. The Reynolds number R is the one from the adiabatic wall case, i.e., the abscissa can be thought of as x/C. These amplification rates were obtained with $\bar{\psi} = \psi_p$ and k so that V_β / V_α is real. The trends of Figs. 7-9 are shown to apply in the entire region of strong crossflow. Wall cooling has a mild stabilizing effect on crossflow instability. This result is true for all the three frequencies 0,1, and 5 kHz examined. From Figs. 7-9, it can be seen that as the crossflow

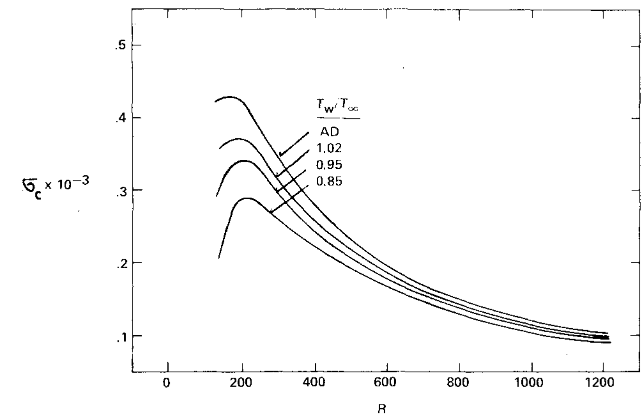

Fig. 8 Variation of growth rates σ_c with different wall temperatures for $F = 1000$ Hz.

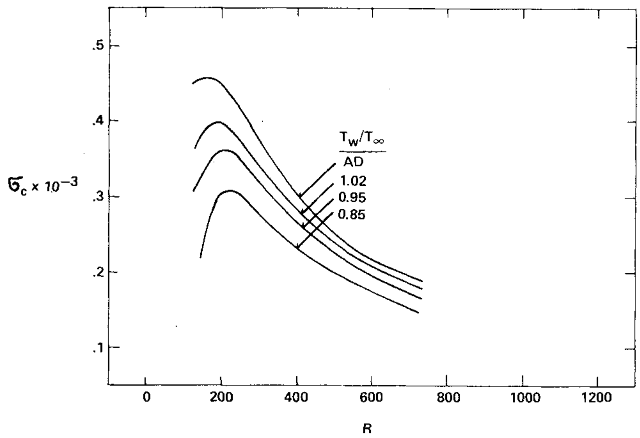

Fig. 9 Variation of growth rates σ_c with different wall temperatures for $F = 5000$ Hz.

strength decreases with increasing R, the stabilizing effect of cooling is reduced.

The effect of wall cooling on the integrated amplification (N factors) for the same region of the wing is shown in Fig. 10. By the end of the region of strong crossflow, the N factor for the coldest wall case is about 75% of the N factor for the adiabatic case. The dashed lines show incompressible calculations. The stabilizing effect of compressibility[20,21] is evident from comparing dashed and solid lines for the same temperature ratio. However, when the wall-to-freestream temperature ratios become less than about 0.95, the incompressible calculations underestimate the stability of the flow for the cases examined. That is, the calculated amplification rates from the incompressible theory are less than the amplification rates calculated from the compressible theory. The reason for this effect can be found in Fig. 4. As the wall cooling ratios decrease, the temperature profiles approach a uniform distribution because T_w/T_e approaches unity. Hence, there is a more uniform distribution of properties inside the boundary layer, and the incompressible and compressible calculations show results that are very close. Therefore the incompressible calculations exaggerate the effect of cooling for the computed temperature ratios because the stabilizing effect of compressibility is reversed at the lower T_w/T_∞.

For the pressure distribution used, there is a region of small crossflow for $0.2 < x/C < 0.5$. Obtaining compressible eigenvalues that satisfy ϕ real becomes expensive in this region. Figures 11 and 12 show N factors for two frequencies (1 and 5 kHz) and for the four different cooling ratios. These results are obtained using the incompressible theory as developed by Srokowski and Orszag.[15] The difference in the spatial am-

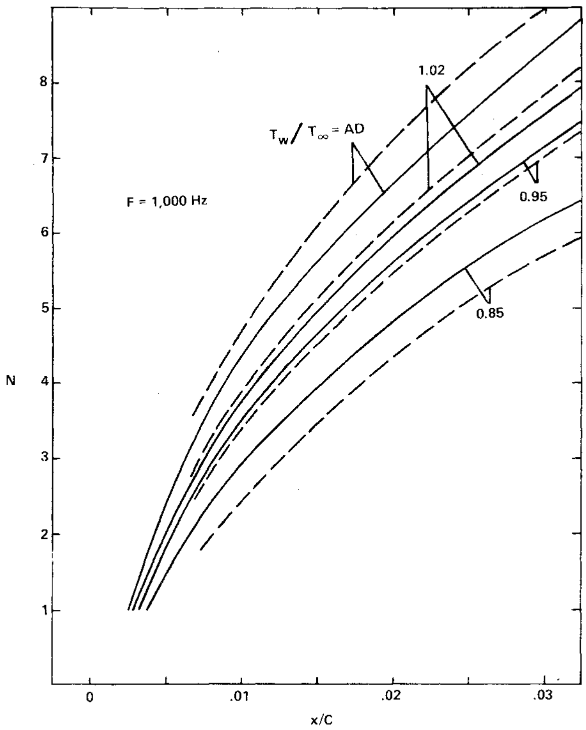

Fig. 10 Growth factors for different wall temperatures and $F = 1000$ Hz; (---) incompressible theory, (—) compressible theory.

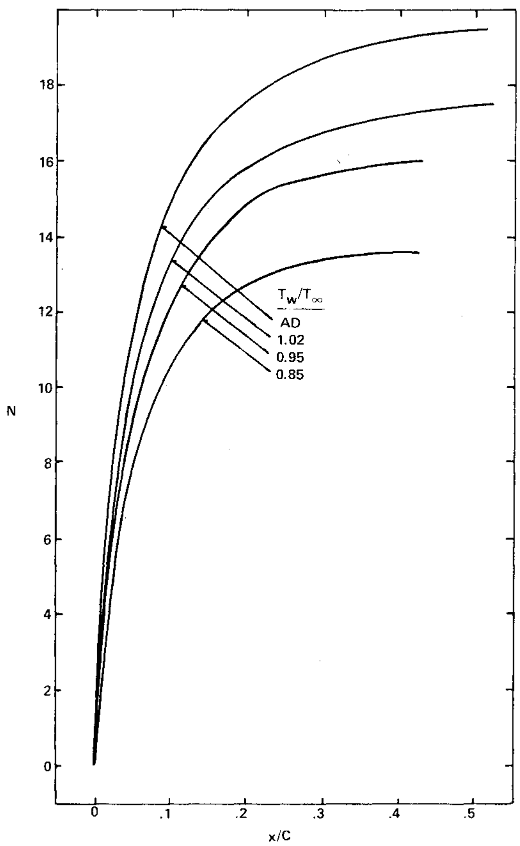

Fig. 11 Growth factors for different wall temperatures and $F = 1000$ Hz; incompressible theory.

plification rates in the direction of ϕ obtained from the temporal amplification rates by using the real part of group velocities,[15] and by using the Nayfeh-Padhye transformation,[17] are about 1% for the cases examined. The incompressible stability calculations use the same boundary-

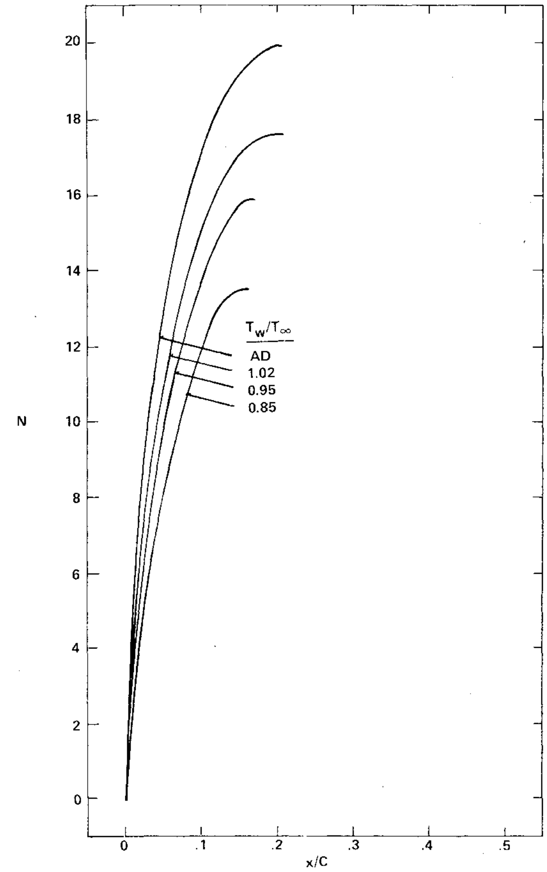

Fig. 12 Growth factors for different wall temperatures and $F = 5000$ Hz; incompressible theory.

layer profiles as do the compressible stability calculations. With the discussion of the previous paragraph about compressible vs incompressible results in mind, the following is observed.

Although the local reduction of amplification rates with cooling is small after $x/C \sim 0.03$, for unstable waves due to crossflow, there is considerable contribution of this reduction to the N factors. The reason is that this small reduction is multiplied by large $\Delta x/C$ values. The N values stop where the local amplification rate becomes negative. Even with the stabilizing effect of cooling exaggerated by the incompressible theory for the cooling ratios used in the computations, the total reduction of N factors is about 30%, as shown in Figs. 11 and 12. These reductions are small compared with the reductions on amplification rates for Tollmien-Schlichting waves with wall cooling shown in Fig. 5. The same conclusion is reached when the results for crossflow instability are compared to the extensive calculations of Ref. 9.

The preceding results show the importance of considering crossflow instability when design studies are made for LFC through wall cooling for a transport with swept wings. Reductions of N factors for unstable crossflow waves with wall cooling will require much smaller wall temperatures than the ones required for comparable reductions for Tollmien-Schlichting waves. If these reductions are impossible for other reasons, like the smallness of the temperature itself as related to the structural integrity of the aircraft skin, then reduction of wing sweep will become necessary. However, the sensitivity of stability calculations to the flow conditions require that each particular case must be studied separately.

V. Conclusions

The laminar stabilities that arise due to crossflow on a swept wing with wall cooling have been investigated, and the following points have been noted:

1) When the condition of real ratio of the complex group velocities is applied to monochromatic waves using temporal stability, it results in a single eigenvalue with maximum temporal amplification rate.

2) When the same condition is applied using spatial stability, the growth in the direction of propagation is independent of the direction of the wavegrowth. The direction of propagation remains almost constant, and the variation of the wavenumber vector both in magnitude and direction is very small. The result is valid for the range of angles of wavegrowth for which eigenvalue convergence was possible.

3) The directions of propagation, the wavenumbers, and the growth rates vary significantly with the chordwise location, indicating strong inhomogeneity in the boundary layer.

4) Cooling the wall stabilizes unstable crossflow waves. However, the stabilization is mild compared with the stabilization of Tollmien-Schlichting waves for the same cooling ratios.

Acknowledgment

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